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THE ACCURACY OF SUBROUTINE CIST

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SUMMARY

A study has been made of the accuracy of the present version of subroutine CIST. This document describes the results of that study.

Most of the specialized logics in CIST, which are designed to deal with special iteration problems, were found to function perfectly. However, there are some problems that CIST cannot solve with 100 percent reliability. This inadequacy is inherent in the basic logic of the present version of CIST.

The accuracy of CIST output depends upon the type of trajectory called for, or, more specifically, the position of trajectory pericynthion and the injection window called for. The RMS pericynthion miss distance of all test cases in this study, which represent a sampling of trajectories encountered in present CIST usage, was a rather large 1335 n. mi. This figure can be reduced with continued development of the empirical trajectory simulation equations.

INTRODUCTION

What CIST Does

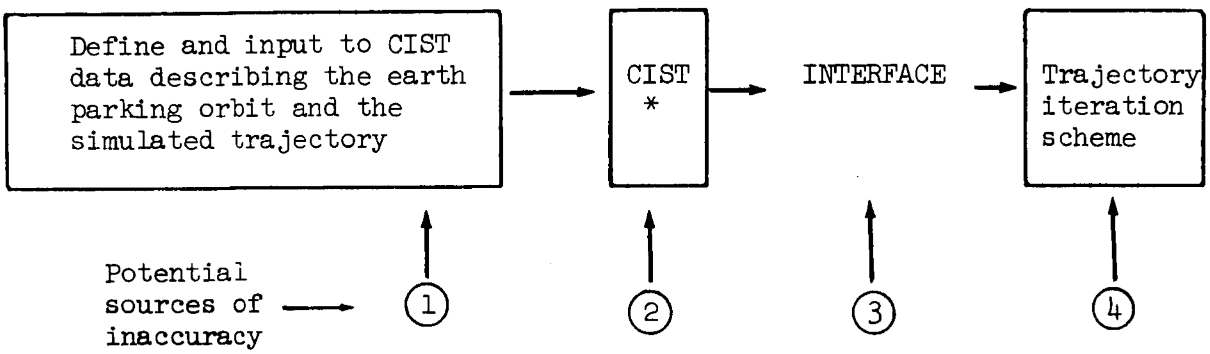
Reference 1 describes an unconventional, efficient method of calculating translunar trajectories. Trajectories calculated by this method are referred to as simulated trajectories. A simulated trajectory combines the accuracy of a numerically-integrated trajectory with the speed of calculation of a patched-conic trajectory.

At present, simulated trajectories are used in mission planning only to provide first guesses to trajectory iteration schemes using conventional, less efficient methods of trajectory calculation. These first-guess logics are based upon the use of subroutine CIST.

CIST defines the coplanar injection into a simulated trajectory from a specified earth parking orbit. Reference 2 describes how CIST works and how it is used in the first-guess logic for the RTCC TLI processor.

Inaccuracies in Present CIST Usage

The following diagram shows in simplified form the logic of a typical CIST application.[a]



It can be seen that there are four potential sources of inaccuracy which can cause nonconvergence or inefficiency in the trajectory iteration. CIST is only one of these four potential sources. Yet for no apparent reason, when the trajectory iterator runs too long or doesn't converge, it is CIST that usually gets all of the blame.

The three sources of inaccuracy, besides CIST, are as follows:

Source (1) - The definition and input of data to CIST. This data describes the earth parking orbit from which TLI occurs, and the simulated trajectory. Typical sources of inaccuracy are misunderstandings of

a. What variables constitute the input data.

b. Units of the input data.

c. The coordinate system in which the input data are defined.

d. How the data are input to CIST.

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[a]For all practical purposes, subroutine UPDATE, which is used in the RTCC TLI first-guess logic, can be considered as being part of the CIST block in this diagram.

Source (3) - The interface between CIST and the trajectory iteration scheme. Inaccuracy usually occurs because of misunderstandings of what variables are put out by CIST, which of these variables should be used to obtain first guesses for the trajectory iteration scheme, and how these variables should be used.

Source (4) - The trajectory iteration scheme. Iteration schemes which cannot converge to an obtainable solution, or which cannot obtain a solution efficiently, are not unheard of.

How to Evaluate the Accuracy of CIST

It has been common practice to evaluate the accuracy of CIST in terms of whether the trajectory iteration scheme converges and how many iterations it takes to achieve convergence. The very gross assumption made in these "evaluations" is that CIST ((2)) is the only source of inaccuracy - that other inaccuracy sources ((1), (3) , and (4)) are nonexistent.

The only meaningful measure of the accuracy of the present version of CIST is the miss distance between the pericynthion of a numerically integrated trajectory with all perturbations propagated directly from CIST output, and the pericynthion position of the simulated trajectory, which is input to CIST.

A study has been made to evaluate the accuracy of CIST in terms of this pericynthion miss distance. This note describes the results of that study.

The only possible source of inaccuracy external to CIST in the study would be the interface between CIST and the trajectory integrator. However, the trajectory state vector (perigee) from which the integrated trajectory was propagated was printed out by both CIST and the integrator program. In every case these state vectors were found to be identical; i.e., no inaccuracies were introduced at this interface.

Sources of Inaccuracy in CIST

When subroutine CIST is called, the iteration logic in CIST will either converge to a solution, or it will not. In either event, an actual solution may or may not actually exist. These four possible situations which can be encountered when CIST is called are shown in the following table:

Four situations	CIST converges to a solution	Does solution actually exist	Comment on situation
1	No	No	Desirable
2	Yes	Yes	Desirable
3	No	Yes	Undesirable
4	Yes	No	Undesirable

The solutions defined by CIST in situation 2 may or may not be accurate. The existence of the undesirable situations, 3 and 4, and inaccurate solutions in situation 2, can be blamed on any one, or combination of, the following four inadequacies in the present version of CIST.

Sources of inaccuracy in the present version of CIST
1. Inaccurately calculated earth parking orbit
2. Inaccurate simulation of coplanar TLI
3. Inaccurate trajectory simulation
4. Inadequate iteration logic

The study of CIST accuracy described in this note is essentially a study of the third and fourth sources of inaccuracy listed above: the trajectory simulation and iteration logic. Evaluations of the inaccuracies introduced by the parking orbit calculation and TLI simulation in CIST are beyond the intended scope of this study.

In all cases, regardless of whether CIST converged or not, no effort was made to prove whether an actual solution actually existed; i.e., integrated trajectories were not iterated to obtain the pericynthion position of the simulated trajectory.

STUDY OF THE ACCURACY OF SUBROUTINE CIST

Outline of the Four Phases of the Study

This study was conducted in four sequential steps:

1. Generation of objectively random input data to CIST.

2. Running CIST and integrating the single translunar trajectories from perigee defined by CIST, to pericynthion.

3. Calculate pericynthion miss distance, from pericynthion of the integrated trajectory to that of the simulated trajectory.

4. Analyze miss distances.

How Input Data to CIST Was Generated: The Lottery

It is very easy to choose input data to CIST such that the resultant pericynthion miss distance of the integrated trajectory will be phenomenally small. For example, when $|i_{vtl}|$ is in the order of several degrees of arc and the pericynthion of the simulated trajectory is at a radius of 1015 n. mi. on the earth-moon axis, miss distance will always be small. (See the table on page 13 for typical results.)

However, this sort of data is not typical of that which is input to CIST in present CIST applications. A study based on "chosen" data of this sort would show CIST to be much more accurate than it actually is in present usage.

To eliminate any subjectivity in choosing the CIST input data in this study, the data cases were chosen by lottery. Literally, each data case was obtained by the random choice of numbered cards from a set of boxes by a different person. Each box represented one of six input variables to CIST which were varied from case to case in this study. Cards in a given box represented a uniform distribution of the given variable over a range which is typical of present CIST usage.

In all test cases, the position of the input earth parking orbit state vector was approximately that of Cape Kennedy (28.5° latitude, -80.6° longitude). Launch azimuth (azimuth at this state vector) and the year and day within which launch time is to be defined by CIST were determined by lottery. In all test cases, the earth parking orbit was circular and had an altitude of 100 n. mi. The beginning of TLI always occurred during the first inertial revolution following launch (the input state vector). The thrust-to-weight ratio at the beginning of TLI was always 0.70. The pericynthion position coordinates (XPC, YPC, and RPC) of the simulated trajectory were determined by lottery.

Each lottery drawing resulted in four different test cases. In each drawing, two values of RPC were chosen, one from a low range (900 to 1100 n. mi.) and the other from a high range (1500 to 4000 n. mi.). Test cases were run with TLI occurring in both the Atlantic and Pacific windows for each value of RPC, all other variables being unchanged. The resulting $|i_{vtl}|$ in one window was usually much larger than that in the other. The four categories of test cases are denoted thusly:

Category of test cases	RPC range	$\lvert i_{vtl} \rvert$
I	low	low
II	low	high
III	high	low
IV	high	high

The test cases were categorized in this way in order to make the results of this study more meaningful. In other words, while a single RMS value of the pericynthion miss distances of all test cases might be a meaningful measure of CIST accuracy in its present usage, this single RMS value would not be a true measure of the accuracy of CIST in doing what it was designed to do.

Much effort has been expended in developing the empirical simulation equations of trajectories having low $|i_{vtl}|$ and RPC in the low range. Going from categories I to IV, the amount of effort which has gone into the simulation equations in representing the trajectories becomes smaller. In fact, the effort spent in getting the simulation equations to accurately represent trajectories in category IV is essentially zero. Thus, in each test case in category IV, CIST (or rather, the trajectory simulation in CIST) is being asked to do something which it was not designed to do. The RMS pericynthion miss distance of category IV should be expected to be larger than that of category I.

The lottery from which the test cases were obtained is illustrated in the following table:

Variable	Range of variable represented on cards	Number of Cards
Year	1968 to 1975, every year	8
Day in year	5 to 365, at 5-day intervals	73
Launch azimuth	70° to 110°, at 5° intervals	9
XPC	15° to -45°, at 5° intervals	13
YPC	10° to -10°, at 5° intervals	5
Low RPC	900 to 1100 nm, at 50 nm intervals	5
High RPC	1500 to 4000 nm, at 500 nm intervals	6

Results of the Lottery: The Input Data Generated

Fourteen persons were asked to participate in the lottery defining the input data to CIST used in this study. The resulting 56 test cases (four per person) are identified by the combination of a guide number and the letter L or H. Odd guide numbers represent test cases with RPC in the low range. Even guide numbers represent test cases with RPC in the high range. The letter L represents test cases with TLI out of that window (Atlantic or Pacific) resulting in the lower value of $|i_{vtl}|$, and the letter H represents test cases having the higher value of $|i_{vtl}|$.

For example, test case 20L would have RPC in the high range with the lower of the two values of $|i_{vtl}|$.

The results of the lottery are indicated in the following table:

Participant	Guide Number	Year	Day	Launch Azimuth	XPC	YPC	RPC
Holmes	1	1973	305	85	-45	-10	1100
	2						2000
Scheffman	3	1974	260	85	15	10	950
	4						3000
Morrey	5	1969	145	90	-35	-5	950
	6						2500
McCaffety	7	1970	285	95	5	-10	1100
	8						4000
Jenness	9	1969	180	90	15	-5	1100
	10						2500
Duncan	11	1971	330	70	10	-10	950
	12						2000
Beck	13	1970	350	90	10	0	1000
	14						1500
Hartung	15	1973	235	90	-5	10	1100
	16						2000
Elk	17	1971	345	80	-20	5	1100
	18						2000
Linbeck	19	1974	305	90	5	5	950
	20						2500
Hagar	21	1974	95	75	-10	-10	1100
	22						4000
Krchnak	23	1973	25	90	-40	-5	1000
	24						2000
Frank	25	1974	50	80	-30	-5	1100
	26						2500
Ernull	27	1968	315	75	0	-10	1000
	28						2000

Results of the Study

In all test cases, CIST converged to a solution. The inaccesible node problem (ref. 2) was not encountered in any of the test cases. This particular type of problem is felt to be the greatest weakness of the present version of CIST. Other types of special problems were encountered in the test cases and the special logics in CIST designed to deal with these problems performed perfectly.

However, in two test cases (9L and 10L) the integrated trajectories did not even get into lunar reference and their pericynthia were never defined. Consequently, these two test cases are not included in the calculations of RMS miss distances. These two test cases, and other test cases of special interest (those having large miss distances), are discussed in the final section of this note.

The tables on pages 10 and 11 show the pericynthion positions and miss distances of the integrated in trajectories in the four categories of test cases.

CATEGORY I - Low RPC and low i_{vtl}

TEST CASE	IVTL	XPC ACHIEVED	YPC ACHIEVED	RPC ACHIEVED	MISS DISTANCE (NM)
1L	-4.96	-63.0663	-12.5371	301.475	818.368
3L	10.17	17.5945	11.0253	1125.937	182.801
5L	-0.02	-47.8747	-2.1006	378.427	587.887
7L	-1.08	6.1312	-10.2045	1158.967	63.048
11L	-8.13	-2.2976	-14.8870	511.895	465.523
13L	0.73	8.4278	-0.4645	907.459	96.471
15L	4.76	-3.1984	10.5121	1220.299	125.952
17L	6.21	-17.9179	4.8604	1240.733	146.979
19L	13.65	3.9366	4.8735	896.241	56.438
21L	13.19	-8.5832	-8.6603	1169.750	79.590
23L	13.60	-33.6500	-5.1079	1309.303	334.087
25L	-6.59	-30.0433	-5.7584	1052.814	49.296
27L	4.29	-0.4687	-9.8155	966.801	34.278

RMS MISS = 332.930

CATEGORY II - Low RPC and high i_{vtl}

TEST CASE	IVTL	XPC ACHIEVED	YPC ACHIEVED	RPC ACHIEVED	MISS DISTANCE (NM)
1H	52.61	-53.6305	51.2734	4329.293	3929.566
3H	-36.55	19.7162	16.7518	1323.466	406.134
5H	-45.04	-35.1176	-15.6003	1066.980	219.738
7H	52.77	8.4806	-18.1998	1363.467	324.434
9H	43.61	20.1521	-17.3259	1518.635	514.883
11H	54.33	9.0767	-17.3792	954.842	123.573
13H	-55.37	8.6374	12.2457	913.658	222.568
15H	-46.67	-1.8716	9.3483	1290.708	201.657
17H	-35.32	-15.1891	3.4579	1394.227	313.729
19H	-26.05	4.6039	6.9588	935.954	35.756
21H	-40.85	-6.3150	-9.9663	1289.266	203.741
23H	-14.29	-32.8197	-5.5096	1373.939	401.631
25H	53.54	-20.7038	15.2931	2054.232	1117.185
27H	-51.36	-0.8151	-12.7049	998.965	49.207

RMS MISS = 1123.883

CATEGORY III - High RPC and low i_{vtl}

TEST CASE	IVTL	XPC ACHIEVED	YPC ACHIEVED	RPC ACHIEVED	MISS DISTANCE (NM)
2L	-5.00	-85.9756	-14.3449	145.629	1891.174
4L	11.51	25.9255	-2.6323	5523.611	2786.004
6L	0.03	-71.7287	-1.6655	122.964	2402.682
8L	-1.23	12.1629	-3.2675	5714.378	1898.714
12L	-8.39	11.3576	-2.6361	1965.228	261.182
14L	0.72	10.7893	-0.6173	1545.991	53.147
16L	4.56	0.5599	6.4817	2518.963	578.530
18L	6.99	-20.4839	4.0767	1826.310	177.132
20L	11.95	4.0500	-5.1005	2275.512	477.786
22L	15.66	0.0228	-5.8778	6122.009	2315.564
24L	-10.96	-30.3028	-6.8728	2718.438	821.965
26L	-6.79	-28.5961	-5.3489	2205.777	300.092
28L	4.06	-11.3440	-10.3860	1070.103	972.527

RMS MISS = 1479.477

CATEGORY IV - High RPC and high i_{vtl}

TEST CASE	IVTL	XPC ACHIEVED	YPC ACHIEVED	RPC ACHIEVED	MISS DISTANCE (NM)
2H	51.89	-64.4992	23.0482	989.759	1366.338
4H	-32.63	24.4034	24.9982	5402.699	2695.561
6H	-38.40	-38.7855	-23.4142	2070.932	857.283
8H	48.60	12.2261	-30.8484	5532.972	2354.867
10H	35.87	22.8186	-26.2304	3977.376	1922.897
12H	52.21	12.8649	-37.2385	2534.904	1191.827
14H	-55.74	10.7699	26.3647	1571.728	704.253
16H	-48.35	0.6453	13.8266	2477.674	544.273
18H	-31.69	-18.3639	3.4516	1979.717	80.704
20H	-29.57	3.1811	24.0928	2347.888	821.161
22H	-35.16	6.7870	-1.2965	7750.726	4169.356
24H	17.48	-31.5641	0.2555	2503.046	634.868
26H	52.77	-20.0479	25.4514	4659.362	2862.124
28H	-53.53	-0.7090	9.1687	1459.264	785.150

RMS MISS = 1863.159

The following table summarizes the RMS miss distance of each category and that of all 54 test cases combined.

Category of test cases	RMS pericynthion miss distance (nm)
I	332.930
II	1123.883
III	1479.477
IV	1863.159
I, II, III, and IV combined	1334.574
Special (See next section)	27.311

It is significant that in all test cases, regardless of how small or large the pericynthion miss distance was, the integrated trajectory went around the correct side of the moon.

Comments on Significant Test Cases

1H and 1L.- It is very doubtful whether an actual solution to test case 1H exists. The angle from pericynthion nadir to the orbit-trajectory plane as defined by the tangency surface in CIST (refs. 1 and 2) is greater than the angle CL; i.e., the trajectory simulation indicates that no such trajectory can exist for so large an $|i_{vtl}|$. In these cases, CIST attempts to give the best possible solution by defining perigee as the point in the orbit-trajectory plane closest to pericynthion nadir. (In fact, the coding of this logic in CIST was prompted by the encounter with test case 1H.)

In test 1L, $|i_{vtl}|$ is much lower than in 1H, so there might be an actual solution 1L. However, the large miss distance in 1L might be due entirely to an inaccurate trajectory simulation. The pericynthion position called for in test cases 1L and 1H is extremely far from the earth-moon axis and very few trajectories of this type were analyzed in developing the trajectory simulation equations in CIST.

To illustrate that the large miss distances in test cases 1L and 1H are caused by extreme $|i_{vtl}|$ and pericynthion position, a special series of test cases was run. In these test cases, pericynthion position was on the earth-moon axis (XPC=YPC=0) at a radius (RPC) of 1015 n. mi. Test cases were run on the even-numbered days throughout the same lunar month in 1973 wherein test cases 1L and 1H occurred. On each of these days, that injection window (Atlantic or Pacific) was used which would give the smaller of the two values of $|i_{vtl}|$. Launch azimuth was always 89°. The results of this special series of test cases, listed in the table below, are in marked contrast to the results of test case categories I, II, III, and IV which were derived from an objectively random lottery.

Special category of test cases

(Trajectories aimed at XPC=YPC=0, RPC=1015)

DAY IN 1973	IVTL	XPC ACHIEVED	YPC ACHIEVED	RPC ACHIEVED	MISS DISTANCE (NM)
304	-5.30	-0.5156	-0.5826	981.438	36.195
306	-4.62	-0.5486	-0.8596	980.635	38.681
308	-4.73	-0.2373	-0.8851	1000.268	21.834
310	-5.81	0.0921	-0.6659	1020.885	13.315
312	-9.29	0.1786	-0.1855	1025.445	11.407
314	-20.14	0.3013	0.9962	1027.337	22.276
316	6.99	0.2030	0.0885	1024.379	10.173
318	5.00	0.0824	0.4332	1018.309	8.495
320	4.62	0.2670	0.6209	1030.510	19.650
322	5.26	0.6090	0.7174	1051.002	39.799
324	7.42	0.6966	0.6906	1055.490	44.198
326	13.35	0.4295	0.3089	1037.766	24.660
328	26.01	0.0596	-1.0789	1008.683	20.101
330	-6.56	-0.1938	0.0193	999.589	15.787
332	-5.06	-0.6296	-0.1245	974.788	41.727

RMS MISS = 27.311

<u>5L and 6L</u>.- It is very doubtful whether solutions actually exist for these two test cases. For a given value of $|i_{vtl}|$, there is an upper limit to the value of |YPC| which can be achieved. In test cases 5L and 6L, $|i_{vtl}|$ is very small while |YPC| is quite large. CIST printed out warning messages in both of these test cases, and a specialized logic was called into use which provides a best possible solution.

<u>9L and 10L</u>.- Of the 56 test cases run in this study, only these two resulted in CIST output which would be completely unusable and misleading to a subsequent trajectory iteration scheme.

It is doubtful whether solutions to test cases 9L and 10L actually exist. The |YPC| is very large for the very small values of $|i_{vtl}|$ in these cases. Warning messages were printed out by CIST and a specialized logic was called into use which provides a best possible solution. However, in these two test cases this special logic failed to function properly. (This special logic did function properly in test cases 5L and 6L.)

In 9L and 10L, when $|i_{vtl}|$ became very small, the node between the vehicle's orbit-trajectory plane and the moon's orbit plane became very erratic. As a consequence, TLI occurred about 180° away from where it should have been. The integrated trajectories propagated from CIST output understandably never got into lunar reference and their pericynthions were never defined.

CONCLUSIONS

There are two types of problems which the present version of CIST is not well equipped to deal with.

One of these problems is that of inaccessible nodes. This problem, described in reference 2, can arise when the parking orbit inclination is less than that of the moon's orbit and the moon is near an extreme declination. These inaccessible node problems can be further categorized as to whether or not a solution (coplanar TLI) actually exists. When a solution actually exists, the special logic designed to deal with this problem in the present version of CIST, is known not to be 100 percent reliable in converging to this solution. (This pessimistic evaluation is based upon the experience of the author and other CIST users; unfortunately, inaccessible node problems were not encountered in the test cases of this study.) If no solution actually exists, the present version of CIST cannot provide a best possible solution. This latter inadequacy is inherent in the basic logic of the present version of CIST.

The other problem which CIST is not able to adequately cope with arises when $|i_{vtl}|$ becomes extremely small such that the node between the planes of the parking orbit and the moon's orbit moves erratically. Problems of this type were encountered in test cases 9L and 10L of this study. (Prior to this study, this type of problem had never been encountered.) The inability to deal with these problems is inherent in the present version of CIST, the logic of which is based upon the orderly, iterative definition of the aforementioned node between the two planes.

In CIST, there are a number of specialized logics designed to deal with special prblems, (Again, see ref. 2.) In this study, all types of these problems were encountered (with the exception of the inaccessible node problem) and the specialized logics performed perfectly in dealing with them.

When CIST does converge to an obtainable solution, or to a best possible solution in the event that an actual solution does not exist, the accuracy of the resulting trajectory is proportional to the amount of effort which has gone into the development of the empirical simulation equations in representing that type of trajectory. Extensive equation development can provide pericynthion miss distances in the order of 28 n. mi. (See page 13.) When there is essentially no development effort, miss distances in the order of 1900 n. mi. should be expected. (See page 11.) The RMS pericynthion miss distance of all types of trajectories encountered in present CIST usage is in the order of 1400 n. mi. This figure can be reduced with continued development of the trajectory simulation equations.

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